MECHATRONIC DESIGN

Lab 2: Introduction to Finite Element (FE) method using CALFEM toolbox (static analysis)

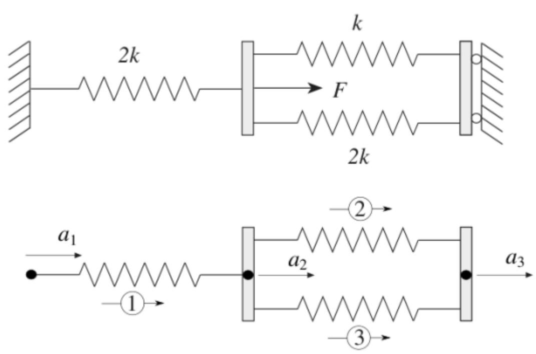
Install CALFEM Toolbox and test its conjunction with MATLAB https://github.com/CALFEM/calfem-matlab

The general procedure in linear FE calculations is carried out for a simple structure. The steps are:

1. Define the model:
    a. Matrix E_{DOF} – number of elements, DOF.
    b. Element coordinates (if needed) – where is the beginning and the end of each element.
    c. Element properties – E [Pa], A [m^2], k [N/m], L [m], ….
2. Generate stiffness element matrices – K^{e1}, K^{e2}, K^{e3}, ….
3. Assemble element matrices into the global system of equations (global stiffness matrices K).
4. Specify boundary conditions (BCs) – forces (force vector), constraints, etc.
5. Solve the global system of equations for the given BCs.
6. Evaluate element displacements, forces, etc.

Example 1:

Consider the system of three linear elastic springs, and the corresponding finite element model. The system of springs is fixed in its ends and loaded by a single load *F*. Evaluate element displacements and corresponding forces.



k = 1500 [N/m]
F = 100 [N]

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% ------ Define Topology matrix
% Edof = [ele1, dof1, dof2
%          ele2, dof1, dof2
%          ...,  ...,  ...]

Edof = [1  1 2
        2  2 3
        3  2 3];

k = 1500;  % N/m
F = 100;   % N

% ------ Initiate stiffness matrix K and load vector f
K = zeros(3, 3);

f = zeros(3, 1);
f(2) = F;
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% ------ Generate stiffness element matrix
ep1 = k;
ep2 = 2*k;

Ke1 = spring1e(ep1)
Ke2 = spring1e(ep2)

% ------ Assemble element matrices into the global stiffness matrix K
K = assem(Edof(1,:), K, Ke2)
K = assem(Edof(2,:), K, Ke1)
K = assem(Edof(3,:), K, Ke2)

% ------ Specify boundary conditions (BCs)
% BC = [dof1   u1
%       dof2   u2
%       ...    ...]

BC = [1 0
      3 0];

% ------ Solve the global system of equations for the given BCs
[a,r] = solveq(K, f, BC)

% ------ Extract element displacement
ed1 = extract_ed(Edof(1,:), a)
ed2 = extract_ed(Edof(2,:), a)
ed3 = extract_ed(Edof(3,:), a)

% ------ Extract element forces
es1 = spring1s(ep2, ed1)
es2 = spring1s(ep1, ed2)
es3 = spring1s(ep2, ed3)
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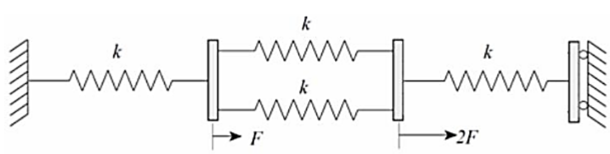
Example 2:

Consider the system of four linear elastic springs, and the corresponding finite element model. The system of springs is fixed in its ends and loaded by a single load *F*. Evaluate element displacements and corresponding forces.

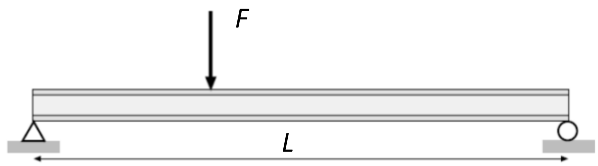


k = 1250 [N/m]
F = 100 [N]

Example 3:

Consider a simply supported beam. The beam is fixed in its ends and loaded by a single load F. Create a finite element model using a two dimensional beam element (***beam2e***). Discretize your model using 10 sequentially connected elements. Evaluate element displacements and corresponding forces. Plot beam displacement and estimate the change in beam deflection depending on where the force *F* is applied. What will happen if force *F* will be applied uniformly across the entire beam?

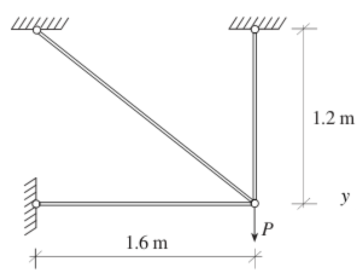
The section forces are evaluated using the function ***beam2s***.



E = 210 [GPa]
L = 10 [m]
A = 0.0453 [m^2]
I = 2510e-8 [m^4]
F = 1000 [N]

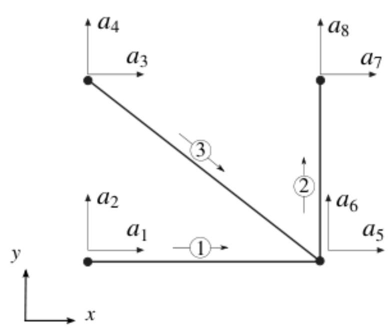
Example 4:

Consider a plane truss consisting of tree bars loaded by a single force *P*.



E = 210 GPa [N]
P = 80 [kN]
A_1 = 0.0006 [m^2]
A_2 = 0.0003 [m^2]
A_3 = 0.0010 [m^2]

The corresponding finite element model consists of three elements and eight degrees of freedom.



The element stiffness matrices Ke1, Ke2, and Ke3 are computed using ***bar2e***. The section forces are calculated using ***bar2s*** from element displacements.

Example 5:

Consider a plane truss, loaded by a single force P. The corresponding finite element model in (a) consists of seventeen elements and eighteen degrees of freedom. A similar model in (b) has fifteen elements and sixteen degrees of freedom. Compare the results of models (a) and (b).

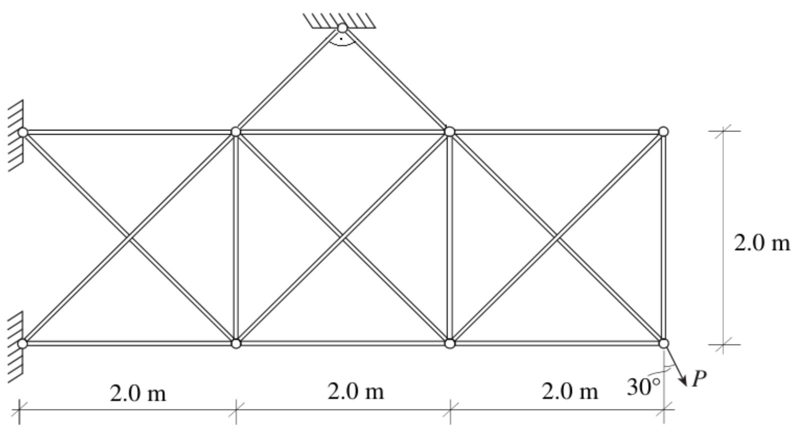
The element matrices Ke are computed by the function ***bar2e***. Normal forces are evaluated using the function ***bar2s***.

E = 210 GPa [N]

A = 0.0025 [m^2]

P = 0.5 [MN]

(a)



(b)